

**RUNNING OF THE COSMOLOGICAL CONSTANT AND ESTIMATE OF ITS VALUE
IN QUANTUM GENERAL RELATIVITY**

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We present the connection between the running of the cosmological constant and the estimate of its value in the resummed quantum gravity realization of quantum general relativity. We also address in this way some of the questions that have been raised concerning this latter generalization and application of the original prescription of Feynman for the formulation of quantum general relativity.

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1. Introduction

As one can see in Refs. [1–9] there has been some controversy about the meaning of a running cosmological constant in quantum field theory. In sum in Ref. [1], the invariance of the physical vacuum energy density under renormalization group action is used to argue that the total response of this quantity to a change in the renormalization scale, μ , is zero, so that it does not actually run. The authors in Ref. [2] argue that, while the total response of the vacuum energy density to a change in such a scale is zero, this still allows for that part of Einstein’s theory that we “see” at low energy to contribute to the implicitly running part of the vacuum energy density, which is then compensated by the dependence on the running scale due to both known contributions and unknown contributions from the possible UV completion of Einstein’s theory. Here, we will present arguments that generally agree with this latter view and with that in Refs. [9, 10], where we use a UV finite approach [11–15] to quantum general relativity developed from an extension of Feynman’s formulation [16, 17] of Einstein’s theory. This we do in the next Section

Having done this, we then show in Section 3 how the running of the cosmological constant and the Newton constant are featured in a first principles estimate of the observed value of the cosmological constant in the Planck scale cosmology scenario of Refs. [23]. Section 4 contains our summary remarks.

^aWe need to stress that the arguments that are given in Ref. [1] do not disagree with those we present here, as we have explained in Ref. [18], when “apples” are compared to “apples”.

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2. Review of the Running of the Cosmological Constant

We begin with a recapitulation of the arguments in Ref. [18]. In this connection, there is one important point of definition. Our discussion will be based on Einstein's equation

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = -8\pi G_N T_{\mu\nu} \quad (1)$$

where $R_{\mu\nu}$ is the contracted Riemann tensor, R is the curvature scalar, $g_{\mu\nu}$ is the metric of space-time, G_N is Newton's constant, $T_{\mu\nu}$ is the matter energy-momentum tensor and Λ is the cosmological constant as we will define it in our discussion. We follow Feynman and expand about flat Minkowski space with the metric representation

$$g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu} \quad (2)$$

where $\kappa = \sqrt{8\pi G_N}$, $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$, so that $h_{\mu\nu}$ is the quantum fluctuating field of the graviton here. If we take the vacuum expectation value of (1), we can move the purely gravitational contribution to the VEV, which arises from the nonlinear part of the “geometric” side of Einstein's equation, to the right-hand side to get

$$\Lambda \eta_{\mu\nu} = -8\pi G_N < 0 | t_{\mu\nu} | 0 > \quad (3)$$

where now

$$< 0 | t_{\mu\nu} | 0 > = < 0 | \left(T_{\mu\nu} + \frac{1}{\kappa^2} (R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu})|_{\text{nonlinear}} \right) | 0 > . \quad (4)$$

From (3), we see that, to any finite order in κ , as the tensor $t_{\mu\nu}$ inside the VEV operation in (4) is conserved in the “flat space” sense [19], it has zero anomalous dimension and this proves that Λ runs because G_N runs. That G_N runs can be inferred immediately from the Dyson resummation of the graviton propagator and the conservation of $t_{\mu\nu}$: in complete analogy with QED, the “invariant charge” then obtains

$$\kappa^2(q^2) = \frac{\kappa^2}{1 + \kappa^2 \Pi(q^2, \mu^2, \kappa^2)} \quad (5)$$

when κ is renormalized at the point μ^2 and $\Pi(q^2, \mu^2, \kappa^2)$ is the respective transverse traceless renormalized proper graviton self-energy function.

The authors in Ref. [1] seem to identify the discussion of the running of the cosmological constant Λ and the discussion of the running of the vacuum energy density. From the definition in Einstein's equation (1), the two attendant quantities are in fact related by a factor of $-8\pi G_N$. Observe that, as the arguments in Ref [1] would show as well that the vacuum energy density does not run, it again would follow that Λ as defined here runs with G_N .

We now agree to call the object analyzed in Ref. [1] by its proper name the vacuum energy density V_{vac} . From (3), the relation

$$\Lambda = -8\pi G_N V_{\text{vac}} \quad (6)$$

holds. The arguments in Ref. [2,9] also seem to be concerned with V_{vac} rather than with Λ as defined in (1). Accordingly, let us now comment further with emphasis regarding the running physics of V_{vac} .

Specifically, we are working with the entire set of degrees of freedom in Einstein's theory as formulated by Feynman, that is to say, we are working with the entire set of degrees of freedom in $h_{\mu\nu}$ for example. By isolating the physics on a given scale Wilson has shown [20, 21] that it is possible to formulate the solution of the theory in the form of scale transformations which evolve the theory from one scale to the next, Wilsonian renormalization group transformations. In Refs. [22–33] Wilson's approach has been pursued in realizing Weinberg's asymptotic safety approach [34] to Einstein's theory. If we thin the degrees

of freedom a la Wilson to those relevant to a given scale μ , as it is done in Refs. [22–33] for example, the effective action at this scale will give the equation such as Einstein’s (1), with perhaps some higher dimensional operators added for a given level of accuracy, but for the theory with the thinned degrees of freedom relevant for the scale μ , with the attendant effective couplings at the scale μ , and this will mean that only that part of the vacuum energy density relevant to the physics on the scale μ will enter into relations such as (3), (6). This means that we have

$$\Lambda(\mu) = -8\pi G_N(\mu)V_{\text{vac}}(\mu) \quad (7)$$

following the general development of Wilson’s renormalization group theory and dropping possible irrelevant operator terms. We conclude that both Λ and V_{vac} run when the theory is solved a la Wilson. This is borne out by the results in Refs. [22–33]. It also supports the arguments in Refs. [2–10].

The basic physics underlying this running of both Λ and V_{vac} is as follows. If we do not thin the degrees of freedom, we can identify a scale [2–8] $\mu = 0$ parameter that corresponds to the vacuum energy density of the universe at arbitrarily long wavelengths and call that $V_{\text{vac-phys}}$ and we can then use Einstein’s equation to identify $\Lambda_{\text{phys}} = -8\pi G_N(0)V_{\text{vac-phys}}$, where $G_N(0)$ is Newton’s constant at zero momentum transfer. These quantities $V_{\text{vac-phys}}$, Λ_{phys} would then be invariant under renormalization and they would not run with changes in the renormalization scale μ . These are the quantities that the authors in Refs. [1] would appear to have in mind when they assert that *the* vacuum energy does not run. On the other hand, following what is done in Refs. [22–33] or following Refs. [35, 36] and implementing Wilsonian renormalization group theory, we are naturally led to effective actions in which degrees of freedom have been thinned on a scale μ and the corresponding values of Λ and V_{vac} , $\Lambda(\mu)$, $V_{\text{vac}}(\mu)$, respectively, for the attendant effective action will run with μ . If a degree of freedom is integrated out of the path integral for the theory, all of its quanta are replaced by their effects on the remaining degrees of freedom. This means that in general V_{vac} runs. We turn next to the implications of such running in estimating the observed value of the cosmological constant as we have done in Ref. [37].

3. Interplay of Running Parameters and Estimating the Observed Value of Lambda

One primary motivation for considering the running of the parameters in the Einstein-Hilbert theory is Weinberg’s suggestion [34] that the theory may be asymptotically safe, with an S-matrix that depends on only a finite number of observable parameters, due to the presence of a non-trivial UV fixed point, with a finite dimensional critical surface in the UV limit. This suggestion has received significant support from the calculations in Refs. [24–33]. Using Wilsonian [20, 21, 35, 38–40] field-space exact renormalization group methods, the latter authors obtain results which support the existence of Weinberg’s UV fixed-point for the Einstein-Hilbert theory. Independently, we have shown [11–15] that the extension of the amplitude-based, exact resummation theory of Ref. [41–59] to the Einstein-Hilbert theory leads to UV-fixed-point behavior for the dimensionless gravitational and cosmological constants. We have called the attendant resummed theory, which is actually UV finite, resummed quantum gravity. We note that causal dynamical triangulated lattice methods have been used in Ref. [60] to show more evidence for Weinberg’s asymptotic safety behavior^b. One can view our results in Refs. [11–15] as helping to put the results in Refs. [24–33, 60] on a more firm theoretical foundation insofar as issues of cut-offs/gauge or lattice artifacts do not arise in our calculations.

Continuing from this latter perspective, we observe that the attendant phenomenological asymptotic safety approach in Refs. [24–33] to quantum gravity has been applied in Refs. [22, 23] to provide an infla-

^bThe model in Ref. [61] realizes many aspects of the effective field theory implied by the anomalous dimension of 2 at the Weinberg UV-fixed point but it does so at the expense of violating Lorentz invariance.

tonless realization^c of the successful inflationary model [63–65] of cosmology : the standard Friedmann-Walker-Robertson classical descriptions are joined smoothly onto Planck scale cosmology developed from the attendant UV fixed point solution. In this way a quantum mechanical solution is obtained to the horizon, flatness, entropy and scale free spectrum problems. Using the new resummed theory [11, 12, 15] of quantum gravity, the properties as used in Refs. [22, 23] for the UV fixed point of quantum gravity are reproduced in Ref. [14] with the bonus of “first principles” predictions for the fixed point values of the respective dimensionless gravitational and cosmological constants. In what follows, we show how the analysis in Ref. [14] can be carried forward [37] to an estimate for the observed cosmological constant Λ in the context of the Planck scale cosmology of Refs. [22, 23]. The quantum field theoretic running of parameters a la Wilson will be seen to play an essential role in the estimate. We comment on the reliability and possible implications of the result, as the estimate will be seen already to be relatively close to the observed value [66–68]. The closeness of our estimate to the experimental value again gives, at the least, some more credibility to the new resummed theory as well as to the methods in Refs. [24–33, 60]^d.

We present the remaining discussion as follows. We start in Section 3.1 with a brief review of the Planck scale cosmology presented phenomenologically in Refs. [22, 23]. Our results in Ref. [14] for the dimensionless gravitational and cosmological constants at the UV fixed point are reviewed in Section 3.2. In Section 3.3, we review the use [37] our results in Section 3.2 in the context of the Planck scale cosmology scenario in Refs. [22, 23] to estimate the observed value of the cosmological constant Λ . We review the use the attendant estimate to constrain SUSY GUTs. We also address consistency checks on the analysis.

3.1. *Planck Scale Cosmology: A Brief Review*

We begin with the Einstein-Hilbert theory

$$\mathcal{L}(x) = \frac{1}{2\kappa^2} \sqrt{-g} (R - 2\Lambda). \quad (8)$$

Here, R is the curvature scalar, g is the determinant of the metric of space-time $g_{\mu\nu}$, Λ is the cosmological constant and $\kappa = \sqrt{8\pi G_N}$ for Newton’s constant G_N . Employing the phenomenological exact renormalization group for the Wilsonian [20, 21, 35, 38–40] coarse grained effective average action in field space, the authors in Ref. [22, 23] have argued that the attendant running Newton constant $G_N(k)$ and running cosmological constant $\Lambda(k)$ approach UV fixed points as k goes to infinity in the deep Euclidean regime. This means that $k^2 G_N(k) \rightarrow g_*$, $\Lambda(k) \rightarrow \lambda_* k^2$ for $k \rightarrow \infty$ in the Euclidean regime.

To make contact with cosmology, one may use a connection between the momentum scale k characterizing the coarseness of the Wilsonian graininess of the average effective action and the cosmological time t . The authors in Refs. [22, 23] use a phenomenological realization of this latter connection, specifically $k(t) = \frac{\xi}{t}$ for some positive constant ξ determined from constraints on physically observable predictions, to show that the standard cosmological equations admit of the following extension:

$$\begin{aligned} \left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2} &= \frac{1}{3}\Lambda + \frac{8\pi}{3}G_N\rho, \\ \dot{\rho} + 3(1 + \omega)\frac{\dot{a}}{a}\rho &= 0, \\ \dot{\Lambda} + 8\pi\rho\dot{G}_N &= 0, \\ G_N(t) &= G_N(k(t)), \Lambda(t) = \Lambda(k(t)). \end{aligned}$$

^cThe authors in Ref. [62] also proposed the attendant choice of the scale $k \sim 1/t$ used in Refs. [22, 23].

^dWe do want to caution against overdoing this closeness to the experimental value.

(9)

Here, we use a standard notation for the density ρ and scale factor $a(t)$ with the Robertson-Walker metric representation given as

$$ds^2 = dt^2 - a(t)^2 \left(\frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right) \quad (10)$$

where $K = 0, 1, -1$ correspond respectively to flat, spherical and pseudo-spherical 3-spaces for constant time t . For the equation of state we take $p(t) = \omega\rho(t)$, where p is the pressure.

From the UV fixed points for $k^2 G_N(k) \equiv g_*$ and $\Lambda(k)/k^2 \equiv \lambda_*$ obtained from their phenomenological, exact renormalization group (asymptotic safety) analysis, the authors in Refs. [22,23] show that the system given above admits, for $K = 0$, a solution in the Planck regime where $0 \leq t \leq t_{\text{class}}$, with t_{class} a “few” times the Planck time t_{Pl} , which joins smoothly onto a solution in the classical regime, $t > t_{\text{class}}$, which coincides with standard Friedmann-Robertson-Walker phenomenology but with the horizon, flatness, scale free Harrison-Zeldovich spectrum, and entropy problems all solved. The solutions are achieved purely by Planck scale quantum physics.

The phenomenological nature of the analyses in Refs. [22,23] is made manifest by the dependencies of the fixed-point results g_*, λ_* on the cut-offs used in the Wilsonian coarse-graining procedure, for example. We note that the key properties of g_*, λ_* used for these analyses are that the two UV limits are both positive and that the product $g_*\lambda_*$ is only mildly cut-off/threshold function dependent. With this latter observations in mind, we review next the predictions in Refs. [14] for these UV limits as implied by resummed quantum gravity(RQG) theory [11,12,15] and show how to use them to predict [37] the current value of Λ . We start the next subsection with a brief review of the basic principles of RQG theory.

3.2. Recapitulation: g_* and λ_* in Resummed Quantum Gravity

We start with the prediction for g_* in Refs. [11,12,14,37]. Given that the theory we use is not very familiar, we review the main steps in the calculation.

As the graviton couples to an elementary particle in the infrared regime which we shall resum independently of the particle’s spin [69], we may use a scalar field to develop the required calculational framework, which we then extend to spinning particles straightforwardly. We follow Feynman in Refs. [16,17] and start with the Lagrangian density for the basic scalar-graviton system:

$$\begin{aligned} \mathcal{L}(x) &= -\frac{1}{2\kappa^2} R\sqrt{-g} + \frac{1}{2} \left(g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - m_o^2 \varphi^2 \right) \sqrt{-g} \\ &= \frac{1}{2} \left\{ h^{\mu\nu, \lambda} \bar{h}_{\mu\nu, \lambda} - 2\eta^{\mu\mu'} \eta^{\lambda\lambda'} \bar{h}_{\mu\lambda, \lambda'} \eta^{\sigma\sigma'} \bar{h}_{\mu'\sigma, \sigma'} \right\} \\ &\quad + \frac{1}{2} \left\{ \varphi_{, \mu} \varphi^{, \mu} - m_o^2 \varphi^2 \right\} - \kappa h^{\mu\nu} \left[\overline{\varphi_{, \mu} \varphi_{, \nu}} + \frac{1}{2} m_o^2 \varphi^2 \eta_{\mu\nu} \right] \\ &\quad - \kappa^2 \left[\frac{1}{2} h_{\lambda\rho} \bar{h}^{\rho\lambda} \left(\varphi_{, \mu} \varphi^{, \mu} - m_o^2 \varphi^2 \right) - 2\eta_{\rho\rho'} h^{\mu\rho} \bar{h}^{\rho'\nu} \varphi_{, \mu} \varphi_{, \nu} \right] + \dots \end{aligned} \quad (11)$$

Here, $\varphi(x)$ can be identified as the physical BEH [70–77] field as our representative scalar field for matter, $\varphi(x)_{, \mu} \equiv \partial_\mu \varphi(x)$, and $g_{\mu\nu}(x) = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}(x)$ where we follow Feynman and expand about Minkowski space so that $\eta_{\mu\nu} = \text{diag}\{1, -1, -1, -1\}$. We have introduced Feynman’s notation $\bar{y}_{\mu\nu} \equiv \frac{1}{2} (y_{\mu\nu} + y_{\nu\mu} - \eta_{\mu\nu} y_\rho{}^\rho)$ for any tensor $y_{\mu\nu}$ ^e. The bare(renormalized) scalar boson mass here is $m_o(m)$ and we set presently the small observed [66–68] value of the cosmological constant to zero so that our quantum graviton, $h_{\mu\nu}$, has zero rest mass. We return to the latter point, however, when we discuss

^eOur conventions for raising and lowering indices in the second line of (11) are the same as those in Ref. [17].

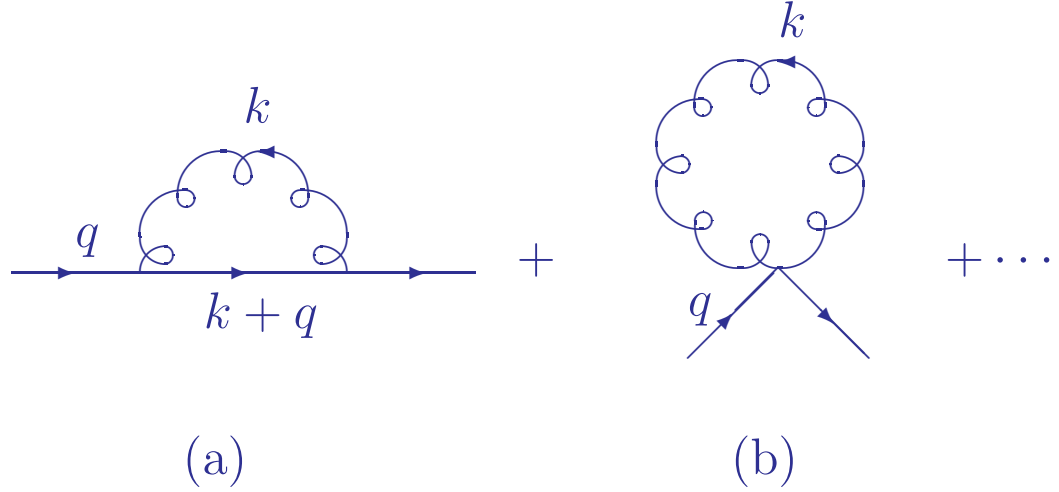


Fig. 1. Graviton loop contributions to the scalar propagator. q is the 4-momentum of the scalar.

phenomenology. Feynman [16, 17] has essentially worked out the Feynman rules for (11), including the rule for the famous Feynman-Faddeev-Popov [16, 78, 79] ghost contribution required for unitarity with the fixing of the gauge (we use the gauge of Feynman in Ref. [16], $\partial^\mu \bar{h}_{\nu\mu} = 0$). For this material we refer to Refs. [16, 17]. We turn now directly to the quantum loop corrections in the theory in (11).

Referring to Fig. 1, we have shown in Refs. [11, 12, 15] that the large virtual IR effects in the respective loop integrals for the scalar propagator in quantum general relativity can be resummed to the *exact* result

$$i\Delta'_F(k) = \frac{i}{k^2 - m^2 - \Sigma_s(k) + i\epsilon} = \frac{ie^{B_g''(k)}}{k^2 - m^2 - \Sigma'_s + i\epsilon} \equiv i\Delta'_F(k)|_{\text{resummed}} \quad \text{for } (\Delta = k^2 - m^2)$$

$$B_g''(k) = -2i\kappa^2 k^4 \frac{\int d^4\ell}{16\pi^4} \frac{1}{\ell^2 - \lambda^2 + i\epsilon} \frac{1}{(\ell^2 + 2\ell k + \Delta + i\epsilon)^2} \quad (12)$$

$$= \frac{\kappa^2 |k^2|}{8\pi^2} \ln \left(\frac{m^2}{m^2 + |k^2|} \right),$$

where the latter form holds for the UV(deep Euclidean) regime, so that $\Delta'_F(k)|_{\text{resummed}}$ falls faster than any power of $|k^2|$ – by Wick rotation, the identification $-|k^2| \equiv k^2$ in the deep Euclidean regime gives immediate analytic continuation to the result in the last line of (12) when the usual $-i\epsilon$, $\epsilon \downarrow 0$, is appended to m^2 . An analogous result [15, 37] holds for $m=0$. Here, $-\Sigma_s(k)$ is the 1PI scalar self-energy function so that $i\Delta'_F(k)$ is the exact scalar propagator. As Σ'_s starts in $\mathcal{O}(\kappa^2)$, we may drop it in calculating one-loop effects. When the respective analogs of $i\Delta'_F(k)|_{\text{resummed}}$ ^f are used for the elementary particles, one-loop corrections are finite. In fact, the use of our resummed propagators renders all quantum gravity loops UV finite [11, 12, 15, 37]. It is this attendant representation of the quantum theory of general relativity that we have called resummed quantum gravity (RQG).

Indeed, when we use our resummed propagator results, as extended to all the particles in the SM Lagrangian and to the graviton itself, working now with the complete theory $\mathcal{L}(x) = \frac{1}{2\kappa^2} \sqrt{-g} (R - 2\Lambda) + \sqrt{-g} L_{SM}^G(x)$ where $L_{SM}^G(x)$ is SM Lagrangian written in diffeomorphism invariant form as explained in Refs. [12, 15], we show in the Refs. [11, 12, 15] that the denominator for the propagation of transverse-traceless modes of the graviton becomes (M_{Pl} is the Planck mass) $q^2 + \Sigma^T(q^2) + i\epsilon \cong q^2 - q^4 \frac{c_{2,eff}}{360\pi M_{Pl}^2}$,

^fThese follow from the observation [15, 69] that the IR limit of the coupling of the graviton to a particle is independent of its spin.

where we have defined $c_{2,eff} = \sum_{\text{SM particles } j} n_j I_2(\lambda_c(j)) \cong 2.56 \times 10^4$ with I_2 defined [11, 12, 15] by $I_2(\lambda_c) = \int_0^\infty dx x^3 (1+x)^{-4-\lambda_c x}$ and with $\lambda_c(j) = \frac{2m_j^2}{\pi M_{Pl}^2}$ and [11, 12, 15] n_j equal to the number of effective degrees of particle j . The details of the derivation of the numerical value of $c_{2,eff}$ are given in Refs. [15]. These results allow us to identify (we use G_N for $G_N(0)$) $G_N(k) = G_N/(1 + \frac{c_{2,eff} k^2}{360\pi M_{Pl}^2})$ and to compute the UV limit g_* as $g_* = \lim_{k^2 \rightarrow \infty} k^2 G_N(k^2) = \frac{360\pi}{c_{2,eff}} \cong 0.0442$.

For the prediction for λ_* , we use the Euler-Lagrange equations to get Einstein's equation as

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = -\kappa^2 T_{\mu\nu} \quad (13)$$

in a standard notation where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$, $R_{\mu\nu}$ is the contracted Riemann tensor, and $T_{\mu\nu}$ is the energy-momentum tensor. Working then with the representation $g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}$ for the flat Minkowski metric $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$ we see that to isolate Λ in Einstein's equation (13) we may evaluate its VEV (vacuum expectation value of both sides). On doing this as described in Ref. [37], we see that a scalar makes the contribution to Λ given by^g

$$\begin{aligned} \Lambda_s &= -8\pi G_N \frac{\int d^4 k}{2(2\pi)^4} \frac{(2k_0^2) e^{-\lambda_c(k^2/(2m^2)) \ln(k^2/m^2+1)}}{k^2 + m^2} \\ &\cong -8\pi G_N \left[\frac{1}{G_N^2 64 \rho^2} \right], \end{aligned} \quad (14)$$

where $\rho = \ln \frac{2}{\lambda_c}$ and we have used the calculus of Refs. [11, 12, 15]. The standard methods [37] then show that a Dirac fermion contributes -4 times Λ_s to Λ , so that the deep UV limit of Λ then becomes, allowing $G_N(k)$ to run, $\Lambda(k) \xrightarrow{k^2 \rightarrow \infty} k^2 \lambda_*$, $\lambda_* = -\frac{c_{2,eff}}{2880} \sum_j (-1)^{F_j} n_j / \rho_j^2 \cong 0.0817$ where F_j is the fermion number of j , n_j is the effective number of degrees of freedom of j and $\rho_j = \rho(\lambda_c(m_j))$. We note that λ_* would vanish in an exactly supersymmetric theory.

For reference, the UV fixed-point calculated here, $(g_*, \lambda_*) \cong (0.0442, 0.0817)$, can be compared with the estimates $(g_*, \lambda_*) \approx (0.27, 0.36)$ in Refs. [22, 23]. In making this comparison, one must keep in mind that the analysis in Refs. [22, 23] did not include the specific SM matter action and that there is definitely cut-off function sensitivity to the results in the latter analyses. What is important is that the qualitative results that g_* and λ_* are both positive and are less than 1 in size are true of our results as well. See Refs. [15, 37] for further discussion of the relationship between our $\{g_*, \lambda_*\}$ predictions and those in Refs. [22, 23].

3.3. Review of an Estimate of Λ and Its Implications

When taken together with those in Refs. [22, 23], the results reviewed here allow us to estimate the value of Λ today. To this end, we take the normal-ordered form of Einstein's equation

$$: G_{\mu\nu} : + \Lambda : g_{\mu\nu} := -\kappa^2 : T_{\mu\nu} : . \quad (15)$$

The coherent state representation of the thermal density matrix then gives the Einstein equation in the form of thermally averaged quantities with Λ given by our result in (14) summed over the degrees of freedom as specified above in lowest order. In Ref. [23], it is argued that the Planck scale cosmology description of inflation gives the transition time between the Planck regime and the classical Friedmann-Robertson-Walker (FRW) regime as $t_{tr} \sim 25 t_{Pl}$. (We discuss in Ref. [37] the uncertainty of this choice of t_{tr} .) We thus start with the quantity $\rho_\Lambda(t_{tr}) \equiv \frac{\Lambda(t_{tr})}{8\pi G_N(t_{tr})} = \frac{-M_{Pl}^4(k_{tr})}{64} \sum_j \frac{(-1)^{F_j} n_j}{\rho_j^2}$ and employ the

^gWe note the use here in the integrand of $2k_0^2$ rather than the $2(\vec{k}^2 + m^2)$ in Ref. [14], to be consistent with $\omega = -1$ [80] for the vacuum stress-energy tensor.

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arguments in Refs. [81] (t_{eq} is the time of matter-radiation equality) to get the first principles field theoretic estimate

$$\begin{aligned}
 \rho_\Lambda(t_0) &\cong \frac{-M_{Pl}^4(1 + c_{2,eff}k_{tr}^2/(360\pi M_{Pl}^2))^2}{64} \sum_j \frac{(-1)^F n_j}{\rho_j^2} \\
 &\quad \times \frac{t_{tr}^2}{t_{eq}^2} \times \left(\frac{t_{eq}^{2/3}}{t_0^{2/3}}\right)^3 \\
 &\cong \frac{-M_{Pl}^2(1.0362)^2(-9.194 \times 10^{-3})(25)^2}{64 t_0^2} \\
 &\cong (2.4 \times 10^{-3} eV)^4.
 \end{aligned} \tag{16}$$

where we take the age of the universe to be $t_0 \cong 13.7 \times 10^9$ yrs. In the latter estimate, the first factor in the second line comes from the period from t_{tr} to t_{eq} which is radiation dominated and the second factor comes from the period from t_{eq} to t_0 which is matter dominated ^h. This estimate should be compared with the experimental result [68]ⁱ $\rho_\Lambda(t_0)|_{\text{expt}} \cong ((2.37 \pm 0.05) \times 10^{-3} eV)^4$.

To sum up, we believe our estimate of $\rho_\Lambda(t_0)$ represents some amount of progress in the long effort to understand its observed value in quantum field theory. Evidently, the estimate is not a precision prediction, as hitherto unseen degrees of freedom, such as those in a high scale GUT theory, may exist that have not been included in the calculation.

Indeed, what would happen to our estimate if there were a GUT theory at high scale? In Ref. [37] we consider the susy SO(10) GUT model in Ref. [83] to illustrate how such theory might affect our estimate of Λ . In this model, the break-down of the GUT gauge symmetry to the low energy gauge symmetry occurs with an intermediate stage with gauge group $SU_{2L} \times SU_{2R} \times U_1 \times SU(3)^c$ where the final break-down to the Standard Model [84–93] gauge group, $SU_{2L} \times U_1 \times SU(3)^c$, occurs at a scale $M_R \gtrsim 2TeV$ while the breakdown of global susy occurs at the (EW) scale M_S which satisfies $M_R > M_S$. What we find is that adding the contributions from the new degrees of freedom for a still viable mass spectrum in this scenario results in a value for the RHS of (16) that has the wrong sign with a significance of many standard deviations. We show [37] that one can resolve this apparent discrepancy either by adding new particles to the scenario, where the known quarks and leptons are doubled with susy partners for the new families that are lighter than the families themselves or by moving the mass of the gravitino to a point near the GUT scale itself, which is $\sim 4 \times 10^{16} GeV$ [83]. Our result for Λ already puts constraints on a class of susy GUT's.

As we explain in Ref. [37], we stress that we actually do not know the precise value of t_{tr} at this point to better than a couple of orders of magnitude which translate to an uncertainty at the level of 10^4 on our estimate of ρ_Λ . We ask the reader to keep this in mind.

We have not mentioned the effect of the various spontaneous symmetry vacuum energies on our ρ_Λ estimate. From the standard methods we know for example that the energy of the broken vacuum for the EW case contributes an amount of order M_W^4 to ρ_Λ . If we consider the GUT symmetry breaking we expect an analogous contribution from spontaneous symmetry breaking of order M_{GUT}^4 . When compared to the RHS of our equation for $\rho_\Lambda(t_{tr})$, which is $\sim -(1.0362)^2 W_\rho / 64 M_{Pl}^4 \simeq \frac{10^{-2}}{64} M_{Pl}^4$, we see that adding these effects thereto would make relative changes in our results at the level of $\frac{64}{10^{-2}} \frac{M_W^4}{M_{Pl}^4} \cong 1 \times 10^{-65}$ and $\frac{64}{10^{-2}} \frac{M_{GUT}^4}{M_{Pl}^4} \cong 7 \times 10^{-7}$, respectively, where we use our value of M_{GUT} given above in the latter evaluation for definiteness. Such small effects are ignored here.

^hThe method of the operator field forces the vacuum energies to follow the same scaling as the non-vacuum excitations.
ⁱSee also Ref. [82] for an analysis that suggests a value for $\rho_\Lambda(t_0)$ that is qualitatively similar to this experimental result.

Concerning the impact of our approach to Λ on the phenomenology of big bang nucleosynthesis (BBN) [94], we recall that the authors in Ref. [23] have already noted that when one passes from the Planck era to the FRW era, a gauge transformation (from the attendant diffeomorphism invariance) is necessary to maintain consistency with the solutions of the system (9) (or of its more general form as given below) at the boundary between the two regimes at the transition time t_{tr} . Requiring that the Hubble parameter be continuous at t_{tr} the authors in Ref. [23] arrive at the gauge transformation on the time for the FRW era relative to the Planck era $t \rightarrow t' = t - t_{as}$ so that continuity of the Hubble parameter at the boundary gives $\frac{\alpha}{t_{tr}} = \frac{1}{2(t_{tr} - t_{as})}$ when $a(t) \propto t^\alpha$ in the (sub-)Planck regime. This implies $t_{as} = (1 - \frac{1}{2\alpha})t_{tr}$. In our case, we have from Ref. [23] the generic case $\alpha = 25$, so that $t_{as} = 0.98t_{tr}$. Here, we use the diffeomorphism invariance of the theory to choose another coordinate transformation for the FRW era, namely, $t \rightarrow t' = \gamma t$ as a part of a dilatation where γ now satisfies the boundary condition required for continuity of the Hubble parameter at t_{tr} : $\frac{\alpha}{t_{tr}} = \frac{1}{2\gamma t_{tr}}$ so that $\gamma = \frac{1}{2\alpha}$. The model in Ref. [23] purports that, for $t > t_{tr}$, one has the time t' and an effective FRW cosmology with such a small value of Λ that it may be treated as zero. Here, we extend this by retaining $\Lambda \neq 0$ so that we may estimate its value. But, with our diffeomorphism transformation between the (sub-)Planck regime and the FRW regime, we can see that, at the time of BBN, the ratio of ρ_Λ to $\frac{3H^2}{8\pi G_N}$ is

$$\begin{aligned} \Omega_\Lambda(t_{BBN}) &= \frac{M_{Pl}^2(1.0362)^2 9.194 \times 10^{-3} (25)^2 / (64t_{BBN}^2)}{(3/(8\pi G_N))(1/(2\gamma t_{BBN})^2)} \\ &\cong \frac{\pi 10^{-2}}{24} \\ &= 1.31 \times 10^{-3}. \end{aligned} \tag{17}$$

Thus, at t_{BBN} our ρ_Λ is small enough that it has a negligible effect on the standard BBN phenomenology. In contrast to what happens in (16), the uncertainty in the value of α does not affect the estimate in (17) because the factors of $\alpha^2 = 25^2$ cancel between the numerator and the denominator on the RHS in the first line of (17).

Turning next to the issue of the covariance of the theory when Λ and G_N depend on time, as we explain in Ref. [37], we follow in Eqs.(9) the corresponding realization of the improved Friedmann and Einstein equations as given in Eqs.(3.24) in Ref. [22]. The more general realization of (9) is given in Eqs.(2.1) in Ref. [23] – our discussions in this Section effectively followed the latter realization. The two realizations differ in the solution of the Bianchi identity constraint: $D^\nu (\Lambda g_{\nu\mu} + 8\pi G_N T_{\nu\mu}) = 0$; for, this identity is solved in (9) for a covariantly conserved $T_{\mu\nu}$ as well whereas, in Eqs.(2.1) in Ref. [23], one has the modified conservation requirement $\dot{\rho} + 3\frac{\dot{a}}{a}(1 + \omega)\rho = -\frac{\dot{\Lambda} + 8\pi\rho\dot{G}_N}{8\pi G_N}$; in (9) the RHS of this latter equation is set to zero. The phenomenology from Ref. [22] is qualitatively unchanged by the simplification in (9) but the attendant details, such as the (sub-)Planck era exponent for the time dependence of a , etc., are affected, as is the relation between $\dot{\Lambda}$ and \dot{G}_N in (9). We note that (9) contains a special case of the more general realization of the Bianchi identity requirement when both Λ and G_N depend on time and in this Section we use that more general realization. We also note that only when $\dot{\Lambda} + 8\pi\rho\dot{G}_N = 0$ holds is covariant conservation of matter in the current universe guaranteed and that either the case with or the case without such guaranteed conservation is possible provided the attendant deviation is small. See Refs. [95–97] for detailed studies of such deviation, including its maximum possible size.

We would note again that the model Planck scale cosmology of Bonanno and Reuter which we use ~~needs~~ more work to remove the type of uncertainties which we just elaborated in our estimate of Λ .

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4. Conclusions

We conclude that the standard methods of the operator field do not support the arguments in Ref. [1] when they are used to argue that Λ and V_{vac} do not run. The arguments in Ref. [1] regarding the renormalization invariance of Λ_{phys} and $V_{\text{vac-phys}}$, defined appropriately, are of course correct.

When the Wilsonian running of the parameters in the Einstein-Hilbert theory is taken into account [22–33, 37], we have shown that, in our resummed quantum gravity realization, we are able to estimate the current value of Λ . That our result is close to observation is at least encouraging.

Note Added

In Refs. [98] it is argued that the physical renormalization group invariant quantities Λ_{phys} and $V_{\text{vac-phys}}$ may actually vary with time for example. This would from our perspective involve dynamics somewhat beyond that which we discuss in this paper.

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